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Residual stress and the instability of axially loaded columns, Colloquium of the International Institute of Welding, Liege, Belgium, (June 1960)

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This work has been carried out as a part of an investigation sponsored jointly by the Column Research Council, the Pennsylvania Department of Highways, the Bureau of Public Roads and the National Science Foundation

Fritz Engineering Laboratory
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Lehigh University
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R É S U M É

Residual stresses influence the strength of columns to an extent dependent upon their magnitude and distribution in the cross section. The basic strength of structural steel columns may be expressed in terms of the tangent modulus. An unfavorable distribution of residual stresses can reduce the tangent modulus quite considerably, lowering the column strength.

Depending upon the size of the individual plates and upon the weld size, the residual stresses in a welded built-up section approach the yield value for the steel; and the column strength may be considerably reduced for the medium slenderness ratios (from 70 to 90). For the welded H-shape this reduction from the yield stress level can be as much as 60%, as compared to 30% for an equivalent rolled shape. Therefore it might be expected that the welded box shape, with beneficial tensile residual stress at the edges, will prove a more economical design where a welded built-up column is desired.

The reduced and tangent modulus concepts, as modified by the presence of residual stress, may be regarded as the upper and lower bounds for column strength. The tangent modulus concept provides a simple, conservative, yet theoretically correct basis for column curves.

I. INTRODUCTION

A research project on the "Influence of Residual Stress on Column Strength and the Mechanical Properties of Rolled Shapes" has been in progress at Lehigh University under the guidance of the Column Research Council. The project was carried out as part of the Council's study of the relationship between material properties and the strength of columns, and the first pronouncement of the Council was its Technical Memorandum No. 1 entitled "THE BASIC COLUMN FORMULA".[1] This memorandum states that the critical or ultimate failure load of a column is given by the equation

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \tag{1}$$

This formula cannot be applied to steel columns if the stress-strain relationship is determined from a small coupon cut from the section. It was shown that residual stresses might account for differences in column strength as much as 30% below that which would be inferred from coupon tests.

Steel column strength depends upon the stress-strain relationship. The latter, in turn, is dependent upon two important factors; these are: (a) the magnitude and distribution of residual stresses, and (b) the basic yield stress level of the material. Therefore, the objectives of the investigation at Lehigh University included a determination of residual stresses in columns, and the development of methods of predicting the influence of these residual stresses on column strength.

The program included tension tests of coupons of the type performed in the mill, compression tests of stub columns (short lengths of full cross-sectional area), and column tests. For the same shapes, residual stresses were measured by the sectioning technique. Theories were developed for predicting column strength, and from the measurements made, it was possible to obtain a correlation with the theory. Maximum strength column formulas could then be written. The program was concerned primarily with rolled wide-flange shapes of ASTM Designation A7 structural steel. References 2, 3 and 4 give a summary of the investigation and contain references to specific aspects of the program on centrally loaded columns. Reference 5 considers eccentrically loaded columns.

It is the purpose of this report to consider the instability of axially loaded columns containing residual stresses. The theory is shown for the general case and then applied to specific cases of engineering application. Experimental results are also included.

II. GENERAL THEORY

For axially loaded columns free from residual stress instability occurs when the Euler load P_e or the tangent modulus load P_t is reached depending on the slenderness ratio and the stress-strain curve of the material.[6,7] The upper and lower limits for column strength are defined by the reduced modulus and the tangent modulus respectively.

The tangent modulus concept assumes that no strain reversal takes place on the convex side of the bent column when it passes from the straight form to the adjacent deflected configuration. The reduced modulus concept assumes that strain reversal of the fibers will take place on the convex side of the bent column when it passes from the straight to the deflected configuration.[6] Since the tangent modulus load gives the bifurcation point at which an initially straight column starts to bend, it can be regarded as the lower limit of column strength. The reduced modulus load, which can only be attained if the column is supported up to the load, may be regarded as the upper limit of column strength since it assumes no further increase in load. The ultimate load of a column has been shown to lie somewhere between these two limits.[6,7] Consideration will now be given to these moduli as applied to columns containing residual stresses.

An initially straight column, either rolled or built-up from plates by welding, will be considered. The derivations to be made will be based on the following assumptions:

1. plane sections remain plane after deformation
2. both the cross section and the residual stress distribution have axial symmetry
3. residual stresses are constant along each fiber of the material
4. each fiber in the cross section of the material follows the same stress-strain law
5. the stress-strain curve for each fiber is completely general. (Specific curves are considered afterwards.)
6. the load is axially applied
7. the ends of the column are pinned.

The cross section of an H-shaped column containing assumed residual strains is shown in Fig. 1(a). Upon loading, the axial strains increase until the bifurcation condition is reached. As with the derivation of the Euler differential equation, equilibrium is considered for the bent position, equating external and internal moments.

From Fig. 1(d) the external moment is

$$M = P \cdot u \tag{2}$$

Because of the symmetry of stresses, the internal moment is due only to the infinitesimal bending stresses $\Delta\sigma$,

Fig. 1(b), which arise at the instant of bifurcation. (The derivation at this stage is not limited to bending about any particular axis.) The internal moment, then, is

$$M = \int_A \Delta\sigma . x . dA \qquad (3)$$

The infinitesimal bending stress $\Delta\sigma$ is a function of the curvature ϕ, the applied average stress σ_{cr}, the residual stress σ_r and the stress-strain curve. For any fiber

$$\Delta\sigma = \bar{E} . \Delta\varepsilon$$

where $\bar{E}$ is a function of the applied and residual stresses, which function would be obtained as a result of assumption 1. $\bar{E}$ varies from point to point across the section. Relating strain to curvature

$$\Delta\sigma = \bar{E} . \phi . e \qquad (4)$$

where e is the distance of the fiber from the position where the infinitesimal bending strain is zero. $\Delta\sigma$ is infinitesimal and is applied to a fiber already at a stress of $f(\sigma_{cr} + \sigma_r)$. Fig. 1(c) shows E_t and $\bar{E}$, and represents the stress strain curve for an individual fiber.

Equations 3 and 4 lead to

$$\begin{aligned} M &= \phi \int_A \bar{E} . e . x . dA \\ &= \phi . E_{mod} . I \end{aligned} \qquad (5)$$

where

$$E_{mod} = \frac{1}{I}\int_A \bar{E}.\ e.\ x.\ dA \tag{6}$$

Equating the moments for equilibrium,

$$M = P.u = \phi.E_{mod}.I$$

which leads to

$$\frac{d^2u}{dz^2} + \frac{P}{E_{mod}I}.u = 0$$

The solution of this equation may be expressed in the form

$$\sigma_{cr} = \pi^2 \frac{E_{mod}}{(\frac{L}{r})^2} \tag{7}$$

Next, consideration will be given to the evaluation of E_{mod}.

a. Tangent Modulus Concept

In accordance with the tangent modulus concept no unloading of fibers is assumed to take place at the instant of bifurcation. Then, from Fig. 1(b) with $e = x + {}^b/2$, Eq. 4 becomes

$$\Delta\sigma = \bar{E}.\phi.(x + \frac{b}{2}) \tag{8}$$

and substitution into Eq. 6 leads to E_i, the effective modulus,

$$E_i = \frac{1}{I}\int_A \bar{E}.x^2.dA \tag{9}$$

and the column equation based on the tangent modulus concept

$$\sigma_{cr} = \pi^2 \frac{E_t}{(\frac{L}{r})^2} \tag{10}$$

Equation 10 was derived by Osgood, reference 8.

b. <u>Reduced Modulus Concept</u>

In accordance with the reduced modulus concept unloading of fibers takes place. For simplicity a rectangular cross section will be assumed. In this case, since the bending entails both an unloading of some fibers and a loading of other fibers in the cross section, the neutral axis shifts to maintain equilibrium. Fig. 2(a) shows these different moduli at a fiber stress of $f(\sigma_{cr} + \sigma_r)$.

The position of the neutral axis can be obtained from the condition of equilibrium of the cross section

$$\int_A \Delta\sigma . dA = \Delta P = 0 \tag{11}$$

Making the assumptions shown in Fig. 2(a) that

$$E(\sigma) = \bar{E} \quad \text{for } \Delta\sigma > 0, \quad \text{loading}$$
$$E(\sigma) = E(0) \quad \text{for } \Delta\sigma < 0, \quad \text{unloading}$$

then, using Figs. 2(a) and (b), with e = y, Eq. 4 becomes

$$\Delta\sigma = \bar{E} . \phi . y \quad \text{for fibers loading}$$

and

$$\Delta\sigma = E(0) . \phi . y \quad \text{for fibers unloading}$$

... (12)

Substitution into Eq. 6 leads to E_{rx}, the reduced modulus modified by residual stresses,

$$E_{rx} = \frac{1}{I_x}\left[\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{0}^{d_1} E\,y^2\,dx\,dy + E(0)\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{0}^{d_2} y^2\,dx\,dy\right] \quad \ldots (13)$$

and the column equation based on the reduced modulus concept

$$\sigma_{cr} = \pi^2 \frac{E_{rx}}{\left(\frac{L}{r}\right)^2} \quad (14)$$

Whenever the stress-strain relationship is non-linear the superposition of stresses no longer holds. A solution would have to satisfy the condition that plane sections remain plane. From this condition the final stress distribution would be obtained, and it would then be theoretically possible to solve Eqs. 9 or 13. Actually, for a stress-strain curve like Fig. 1(c), the solution is quite involved. However, when the stress-strain relationship may be idealized as shown in Fig. 3, the solution may be obtained.

III. IDEALIZED STRESS-STRAIN RELATIONSHIP

A comparatively simple solution for column instability may be obtained when every fiber in the cross section has an idealized elastic-plastic stress-strain relationship, Fig. 3.

The stress-strain curve is idealized so that

$$E(\sigma) = E \qquad \text{for } \sigma < \sigma_y$$

and

$$E(\sigma) = 0 \qquad \text{for } \sigma = \sigma_y \tag{15}$$

a. <u>Tangent Modulus Concept, H-section</u>

Using Eq. 15, Eq. 9 becomes

$$E_i = \frac{E}{I}\int_{A_e} x^2\,dA = E\frac{I_e}{I} \tag{16}$$

where A_e and I_e are the area and the moment of inertia of the portion of the section that is still elastic. Realizing that the yielded portion of a structural shape offers no additional resistance to bending,[9] the buckling strength is then a function of the moment of inertia of the elastic part. Equations 10 and 16 give the critical stress for a pin-end column as

$$\sigma_{cr} = \pi^2 \frac{E\frac{I_e}{I}}{\left(\frac{L}{r}\right)^2} \qquad (17$$

The solution of Eq. 17 to obtain the column curve requires the function relating I_e to σ_{cr}. This can be carried out by either of two methods: one is based on the assumed or measured residual stress distribution in the cross section; the other makes use of the stress-strain relationship obtained from a stub column test.

Residual Stress Method

The process of yielding in the cross section due to the load will be dependent upon the residual stress distribution. Considering a H-section, the following equations are based on the assumption of a parabolic distribution which is generally the case for rolled H-shapes of structural steel.[10] This distribution is given by

$$\sigma_{rx} = \left(\frac{2x}{b}\right)^2 \cdot (\sigma_{rc} - \sigma_{ro}) + \sigma_{ro}$$

$$\sigma_{ry} = \left(\frac{2y}{h}\right)^2 \cdot (\sigma_{ro} - \sigma_{rw}) + \sigma_{rw} \qquad (18)$$

The terms are defined in Fig. 4, which also shows the residual stress distribution and the yielded portions of the cross section.

The average stress on the yielded cross section is

$$\sigma_{ave} = \sigma_y - \sigma_{rx_o} \frac{A_e}{A} - \frac{4t}{A} \int_{x_o}^{\frac{b}{2}} \sigma_{rx}\, dx - \frac{2w}{A} \int_{0}^{y_o} \sigma_{ry}\, dy \qquad \ldots (19)$$

where x_o and y_o define the yielded portions. The relationship between x_o and y_o follows from $\sigma_{rx_o} = \sigma_{ry_o}$ and

$$\left(\frac{2y_o}{h}\right)^2 - \left(\frac{2x_o}{b}\right)^2 \cdot \frac{\sigma_{rc} - \sigma_{ro}}{\sigma_{ro} - \sigma_{rw}} = 1 \qquad (20)$$

The effective moments of inertia are

$$I_{ex} = A_w \cdot \frac{h^2}{12}\left[1-\left(\frac{2y_o}{h}\right)^3\right] + A_f\left(\frac{d+h}{4}\right)^2 \cdot \left(\frac{2x_o}{b}\right)$$

$$I_{ey} = A_f \cdot \frac{b^2}{12}\left(\frac{2x_o}{b}\right)^3 \qquad \ldots (21)$$

The procedure for solution of the column equation is as follows: an assumption for x_o is made, and then y_o may be obtained from Eq. 20. These values are used in Eqs. 19 and 21 from which Eq. 17 may be solved for L/r.

The limiting condition for which yielding may occur only in the flanges or only in the web may be derived from the above Eqs. 19, 20 and 21. In such cases, no assumption is required for the shape of the residual stress distribution, the equations being independent of them.

"Stub Column" Method

When the residual stress distribution is not available, computation of column curves may be carried out making use of a stress-strain curve from a stub column test. It is comparatively simple if yielding in the web is neglected. The stress-strain curve obtained from a stub column test reflects the presence of the residual stresses. Its length is sufficiently small to prevent failure as a column, but long enough to contain the same residual stress pattern that exists in the column itself. The stub column curve shows a stress-strain relationship for the complete cross section, and the proportional limit is reached when $\sigma = \sigma_y - \sigma_r$, Fig. 3. For convenience the distribution of yielding in the cross section may be considered to be of the following three categories: yielding in flanges only, yielding in web only, and yielding in both web and flanges simultaneously.

For yielding in the flanges only and assuming that the web does not yield, or that when it does, the flanges have completely yielded, then[11]

$$E_{ix} = E\frac{I_{ex}}{I_x} = \frac{AE_t - \frac{2}{3}A_w E}{A_f + \frac{A_w}{3}} \quad (\approx E_t)$$

$$E_{iy} = E\frac{I_{ey}}{I_y} = E\left[\frac{AE_t}{A_f E} - \frac{A_w}{A_f}\right]^3 \qquad \ldots (22)$$

As has been shown by test[2,10] the great majority of H-shapes satisfy these conditions. Furthermore, the influence of web yielding is not too pronounced.

With the above simplifications, the measurement of the tangent modulus at any chosen stress on the stub column curve enables direct solution of Eq. 17.

Equations for the other two categories of cross-section yielding are given in reference 12.

b. <u>Tangent and Reduced Modulus Concepts, Rectangular Section</u>

If an axially loaded column of rectangular cross section is considered, then it is possible to make a quantitative comparison of the tangent and reduced modulus concepts.

For a rectangular shape, $A_w = 0$, and the strong- and weak-axis directions are interchanged from those for the H-section. These axes are shown in Fig. 5(a). A parabolic distribution is assumed for the residual stress, such that

$$\sigma_r = \sigma_{ro}\left(1 - \frac{12}{d^2}\cdot y^2\right) \tag{23}$$

Since there is no web, the assumptions for Eq. 22 hold true, so that, with the tangent modulus concept

$$\begin{aligned} E_{ix} &= E\left(\frac{E_t}{E}\right)^3 \\ E_{iy} &= E_t \end{aligned} \tag{24}$$

It may be shown that(9,11)

$$E_t = E\frac{A_e}{A} \tag{25}$$

so that Eqs. 24 become

$$\begin{aligned} E_{ix} &= E\left(\frac{2y_o}{d}\right)^3 \\ E_{iy} &= E\left(\frac{2y_o}{d}\right) \end{aligned} \tag{26}$$

The average stress* in the column above the proportional limit for the stub column stress-strain curve, that is, at a level where some yielding has taken place, Fig. 5(a), is

$$\begin{aligned}\sigma_{cr} &= \sigma_y - \frac{A_e}{A}\cdot\sigma_{ry_o} - \frac{2b}{A}\int_{y_o}^{\frac{d}{2}} \sigma_r \, dy \\ &= \sigma_y + 2\,\sigma_{ro}\left(\frac{2y_o}{d}\right)^3 \qquad (27)\end{aligned}$$

$$\text{for } \sigma \geqslant \sigma_y + 2\sigma_{ro}$$

which, by substitution into Eqs. 26 leads to final expressions for the effective moduli

$$\begin{aligned}E_{ix} &= E\,\frac{\sigma_y - \sigma_{cr}}{2\,\sigma_{ro}} \\ E_{iy} &= E\left(\frac{\sigma_y - \sigma_{cr}}{2\,\sigma_{ro}}\right)^{\frac{1}{3}}\end{aligned} \qquad (28)$$

With the reduced modulus concept Eq. 13 reduces to

$$E_{rx} = \frac{Eb}{I_x}\left[\int_0^{y_o}\left(y + \frac{d}{4} - \frac{y_o}{2}\right)^2 dy + \int_0^{\frac{d}{2}}\left(y - \frac{d}{4} + \frac{y_o}{2}\right)^2 dy\right] \qquad \ldots\ (29)$$

according to the detail given in Fig. 5(b). Figure 5(b) shows the centroidal axis N, and N', the neutral axis shifted from N. It will be noted that the loading stresses of the reduced modulus concept do not act over the complete cross section which is defined by d'_1 and d'_2. From axial equilibrium of the

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* The sign convention is the usual one: compressive stresses negative, tensile stresses positive. Consistent with this convention, negative values are to be substituted for σ_y.

infinitesimal bending stresses, and since $E_t = E(o) = E$ for the loading and unloading of stresses, then $d_1' = d_2' = \frac{d}{4} + \frac{y_o}{2}$.

Equations 27 and 29 finally lead to

$$E_{rx} = \frac{E}{8}\left(1 + \left(\frac{\sigma_y - \sigma_{cr}}{2\sigma_{ro}}\right)^{\frac{1}{3}}\right)^3$$
$$E_{ry} = E\left(\frac{\sigma_y - \sigma_{cr}}{2\sigma_{ro}}\right)^{\frac{1}{3}} = E_{iy} \qquad (30)$$

Figure 6(b) shows a plot of Eqs. 28 and 30 for $\sigma_y = 40$ ksi, $\sigma_{ro} = 10$ ksi and $\sigma_{rc} = 20$ ksi, with the stress-strain curve shown in Fig. 6(a). Equations 10 and 14 give the solution for the column curves and these have been plotted in Fig. 6(c). It may seen that the ultimate column strengths calculated from the reduced modulus concept are upper bounds, and those calculated from the tangent modulus concept are lower bounds. It would be expected that the reduced modulus and tangent modulus concepts also define the upper and lower limits respectively for H-shaped columns containing residual stresses.

Similar equations for the modified moduli may be obtained for stress-strain relationships other than idealized. Studies not yet published show that for the logarithmic curve the modified reduced modulus was again greater than the effective modulus.

IV. TEST RESULTS AND COLUMN CURVE APPROXIMATIONS

Referring to Fig. 6(c) it will be noticed that the curve for buckling in the "weak" direction is approximately parabolic in shape and for buckling in the "strong" direction the curve may be approximated by a straight line. This is for the tangent modulus concept. This is also a good approximation for rolled shapes,(13) except that the "strong" and "weak" axes are interchanged.

Figure 7 shows the column curve approximations, as well as the results of column tests on rolled H-shapes varying in size from 4WF13 to 14WF111. All data has been adjusted to the same basic values of σ_y, $\bar{E}$ and σ_p for all the tests. (The determination of σ_p reflects the evaluation of the measurement of compressive residual stresses at the flange tips of H-shapes varying from 4" to 36" in depth.) It may be seen that curves for column strength based on the tangent modulus method modified by the presence of residual stress reflect actual conditions and afford a realistic basis for the development of design curves.

V. BUILT-UP MEMBERS

In a pilot investigation it was shown that riveted built-up columns were stronger than comparable rolled H-shapes.(14) This is because the residual stress magnitude and distribution was more favorable. Welded built-up columns, on the other hand, have very high residual stresses, particularly tensile residual stresses. For H-shaped members the compressive residual stresses may also be high. The high residual stresses are due to the welding which sets up thermal and residual stresses due to the differential cooling effect.[10] The magnitude and distribution of welding residual stresses is markedly influenced by the geometry of the cross section, and to a lesser extent by the material and thermal properties of the steel. Figure 8 shows residual stresses in a welded H-shaped member. As would be expected, tensile stresses in the vincinity of the weld approach the yield point. Compressive stresses at flange tips were about 20 ksi.

The results of pilot tests, W1, W2 and W3 (Fig. 9) indicated that the strength of welded built-up columns could be predicted satisfactorily, knowing the distribution of residual stresses.

It is evident from a comparison of Figs. 7 and 9 that these particular welded H-shaped columns showed a greater reduction in column strength than the corresponding rolled shapes. This is due mainly to the welding process which causes very high compressive residual stresses to be set up on the flange tips, stresses which may be twice as high as those in the comparable rolled shape. The higher compressive residual stresses infer that more of the section reaches the yield stress at a lower load, so that less of the section is capable of resisting bending, the yielded section playing no further role in resisting bending.[9] Other shapes, however, may well show a quite different result. For example, a welded box shape, edge welded from four plates, has tensile residual stresses formed at the edges. It would be expected that these stresses would enable the shape to resist bending longer, although this would depend on the residual stress distribution and hence on the plate and weld sizes. The general conclusion may be drawn that certain shapes will be stronger and more economical than others, and investigations are continuing on this line.

VI. SUMMARY

1. A solution for column instability can be found for axially loaded columns containing an axially symmetric residual stress distribution. The solution may be in terms of the tangent modulus concept of column buckling, or in terms of the reduced modulus concept. (Section II.)

2. The strength of axially loaded columns may be expressed in terms of the tangent modulus E_t, (Fig. 7). This modulus depends upon the state of residual stress in the member, which stress may be introduced during fabrication operations.

3. For columns containing residual stresses, the reduced and tangent modulus concepts give upper and lower limits for column strength, (Fig. 5). This has been shown for the rectangular cross section, and it would be expected that this would be true for any cross section.

4. Columns built-up by welding may contain tensile residual stresses close to the yield point, (Fig. 8). The compressive residual stresses may be higher or lower than those that form due to cooling, depending on the geometry of the cross section. Although tests of H-shaped welded members exhibit a strength that is comparatively less than that of a corresponding H-shape, (Fig. 9), it might be expected that welded columns of box cross section would have a strength comparable to that of the corresponding rolled member.

VII. NOMENCLATURE

A	cross-sectional area
A_e	area of cross section that is elastic
A_f	area of both flanges of a WF shape
A_w	web area
b	width of H-shape
d	depth of H-shape and of rectangular cross section

d_1, d_2	distances from neutral axis
e	fiber distance from axis of bending
E	Young's modulus of elasticity
$E_t = E(\sigma)$	tangent modulus
$\bar{E}$	tangent modulus at the fiber stress
E_i	effective modulus, tangent modulus concept
E_{rx}	modified reduced modulus, reduced modulus concept
f	function of ...
I	moment of inertia
I_e	moment of inertia of the unyielded (elastic) part
KL/r	effective slenderness ratio
L/r	slenderness ratio
M	moment
P	applied axial load
P_e	Euler buckling load for pin-end column
P_t	tangent modulus load
P_y	axial load corresponding to yield stress across entire section
u	deflection of column from the straight position
x,y	directions for flexural axes
σ	stress
σ_{cr}	applied average maximum stress on a column
σ_r	residual stress
σ_{rc}	residual stress at flange tips
σ_{rw}	residual stress at web center

σ_{ro} residual stress at flange centers

σ_y yield stress level, the average stress in the plastic range

$\Delta\sigma$ infinitesimal bending stress

ϕ curvature.

VIII. ACKNOWLEDGEMENTS

This report presents a part of the theoretical and experimental studies made on a research program on the influence of residual stress on column strength, carried out at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, of which William J. Eney is Director.

The Pennsylvania Department of Highways and the Bureau of Public Roads, the National Science Foundation, and the Engineering Foundation through the Column Research Council jointly sponsor the research program.

A Column Research Council committee under the chairmanship of John A. Gilligan has provided valuable guidance to the project.

Acknowledgement is due also to those other investigators at Fritz Laboratory to whose work reference is made throughout the report.

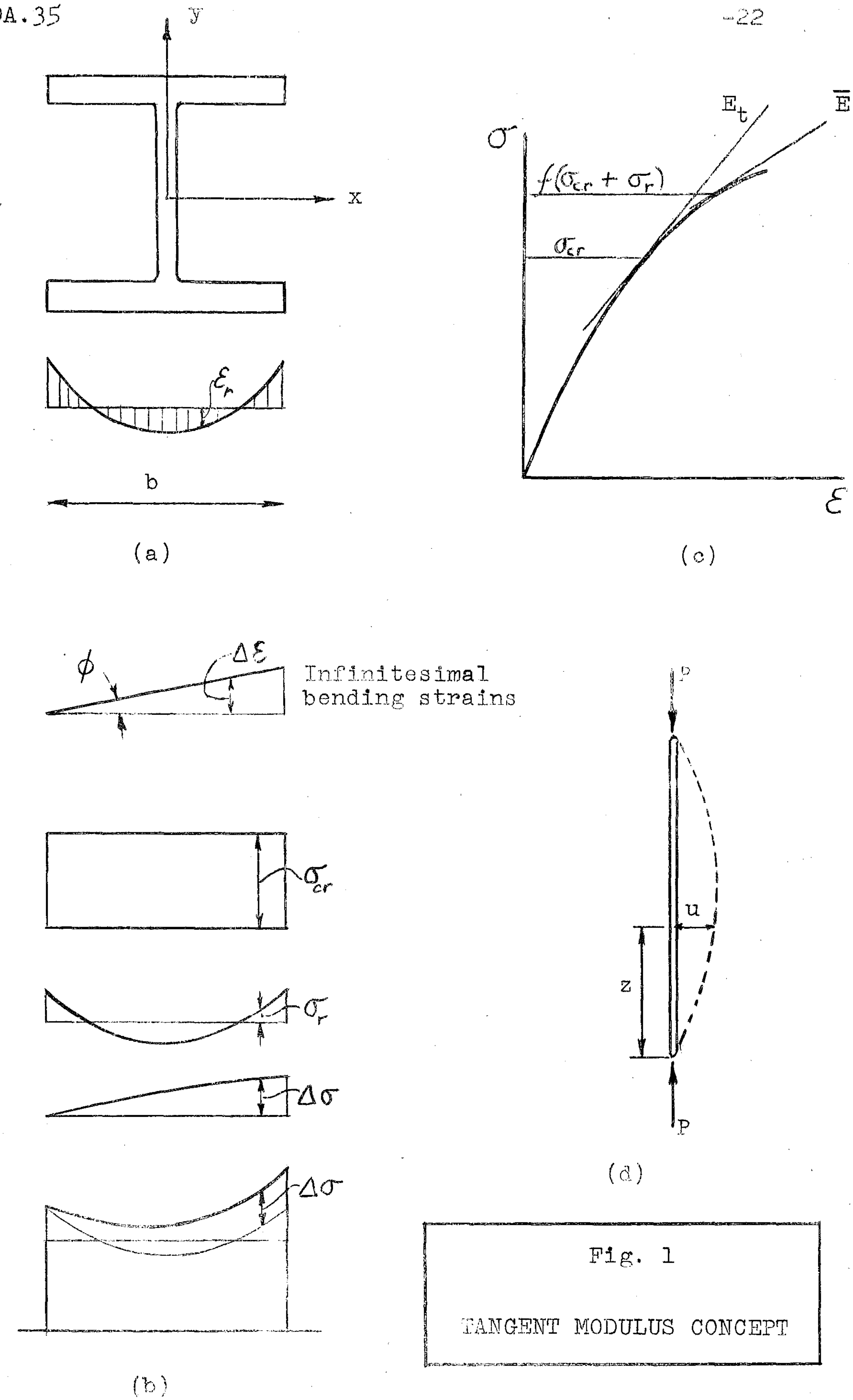
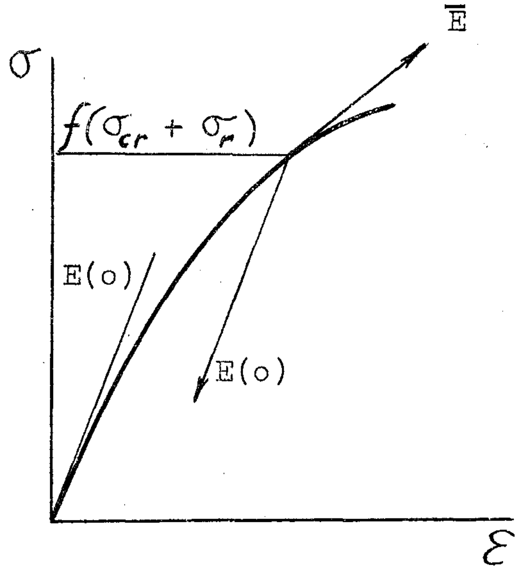
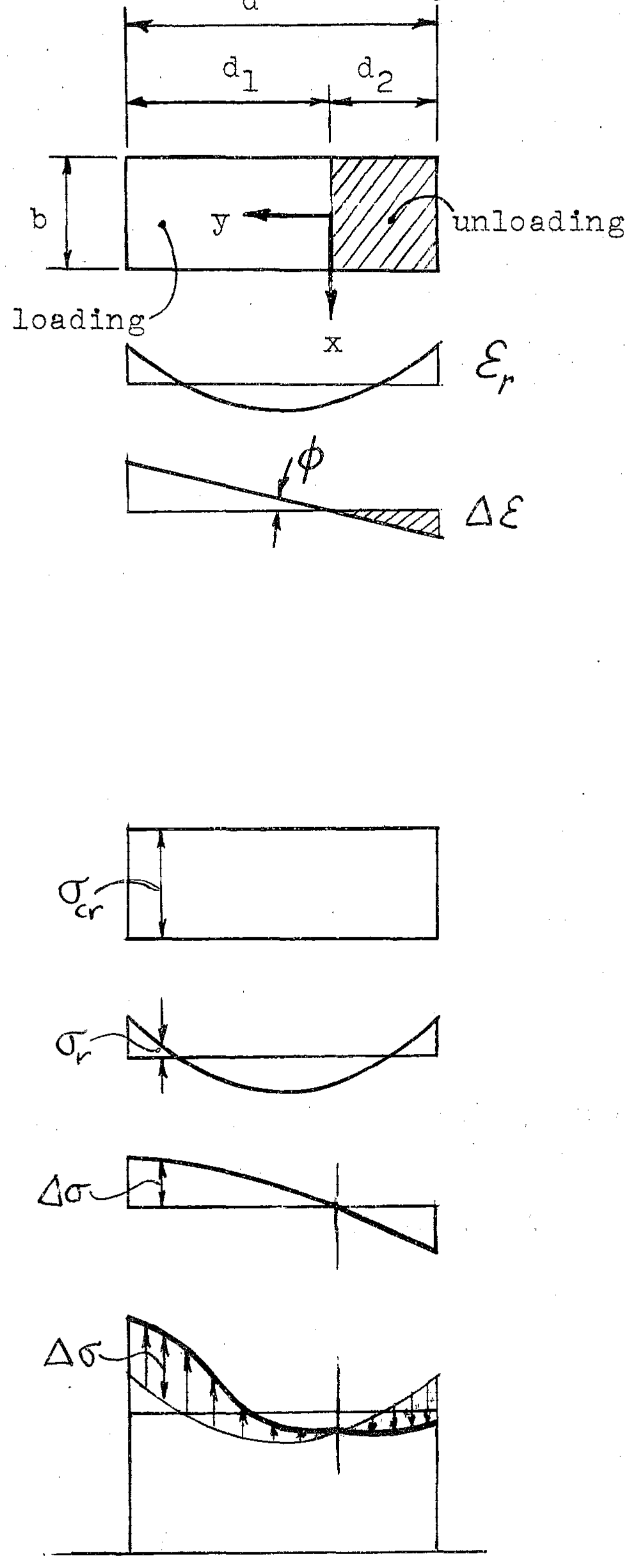


Fig. 1

TANGENT MODULUS CONCEPT



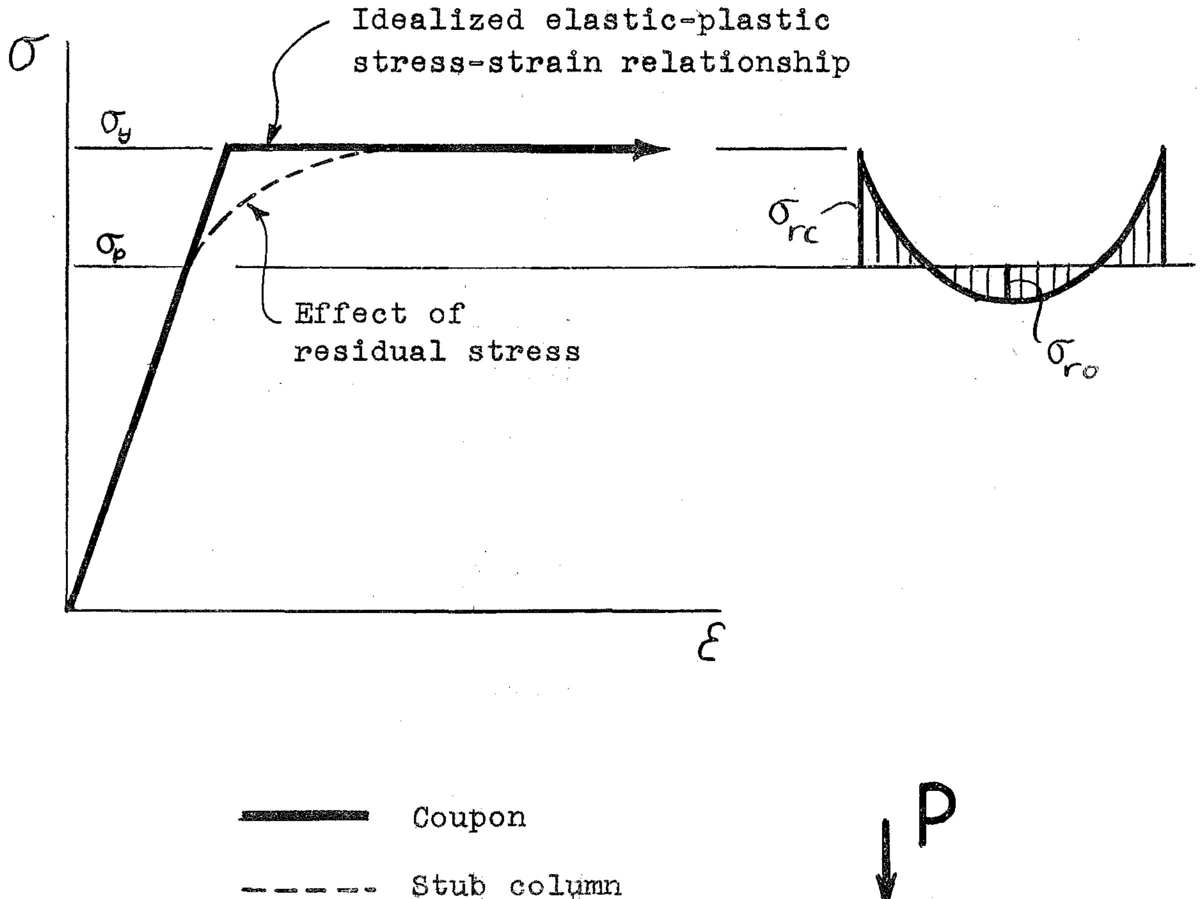
(a)



(b)

Fig. 2

REDUCED MODULUS CONCEPT



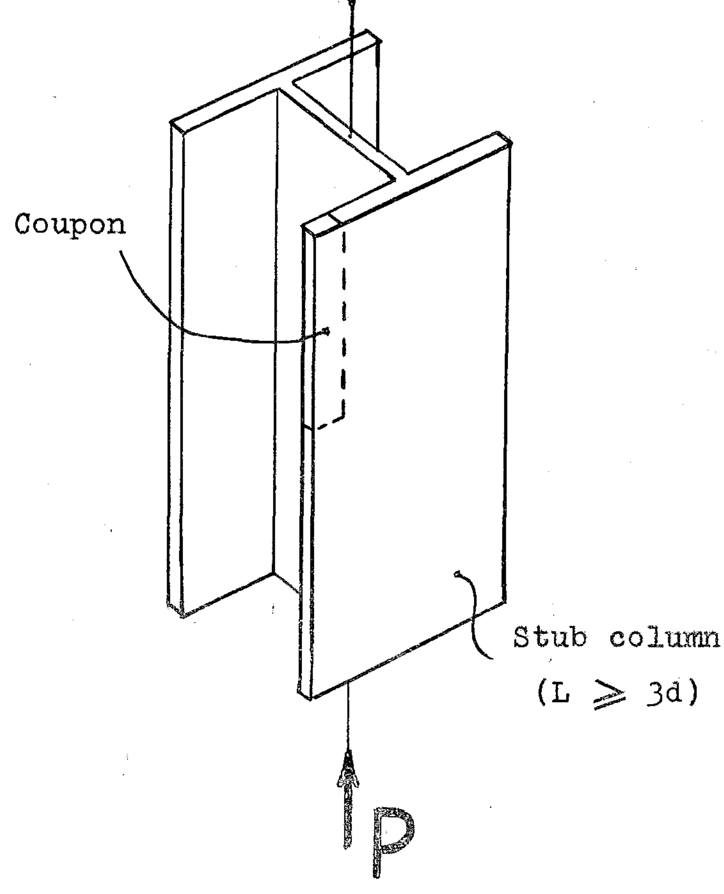


Fig. 3

IDEALIZED STRESS-STRAIN RELATIONSHIP

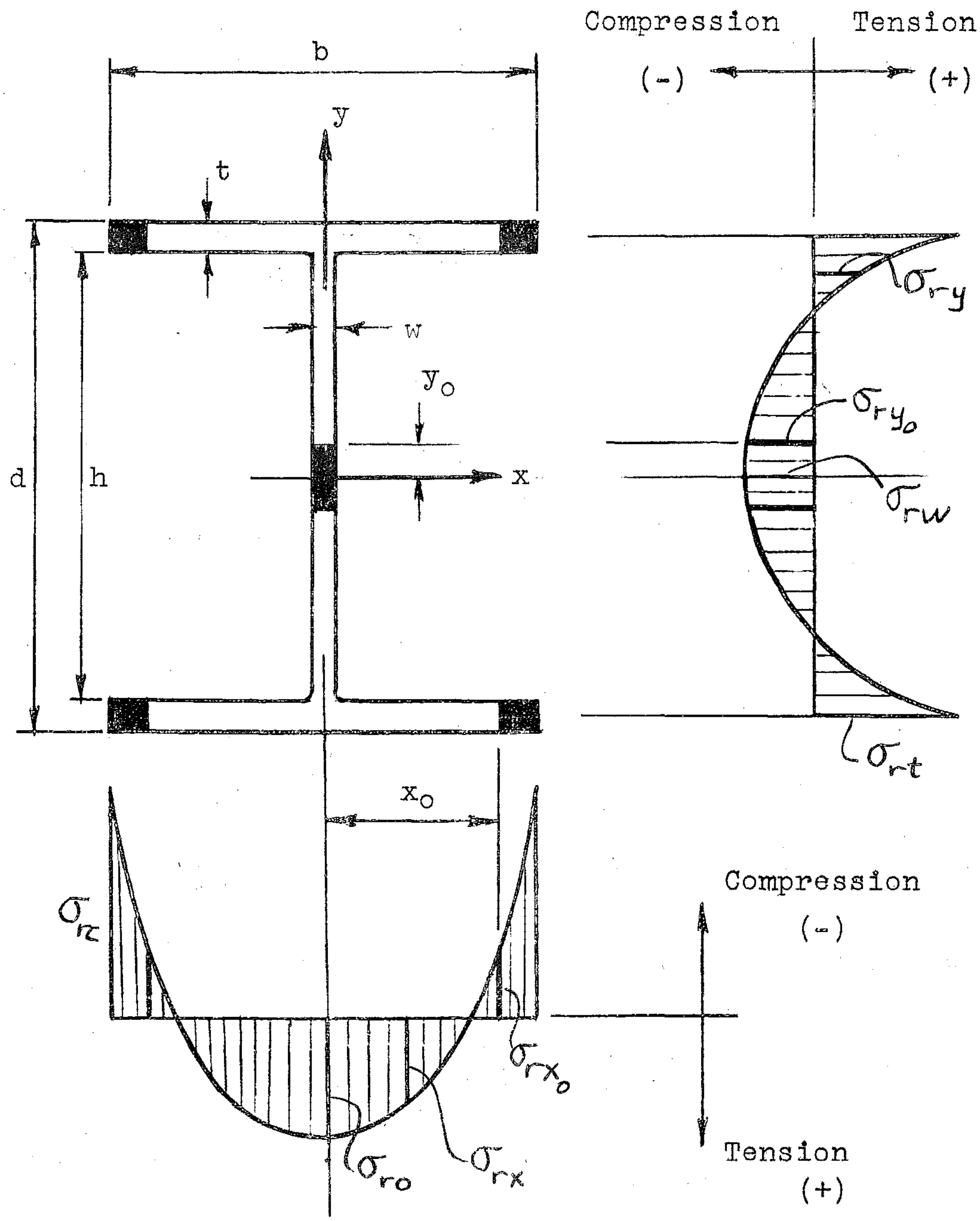
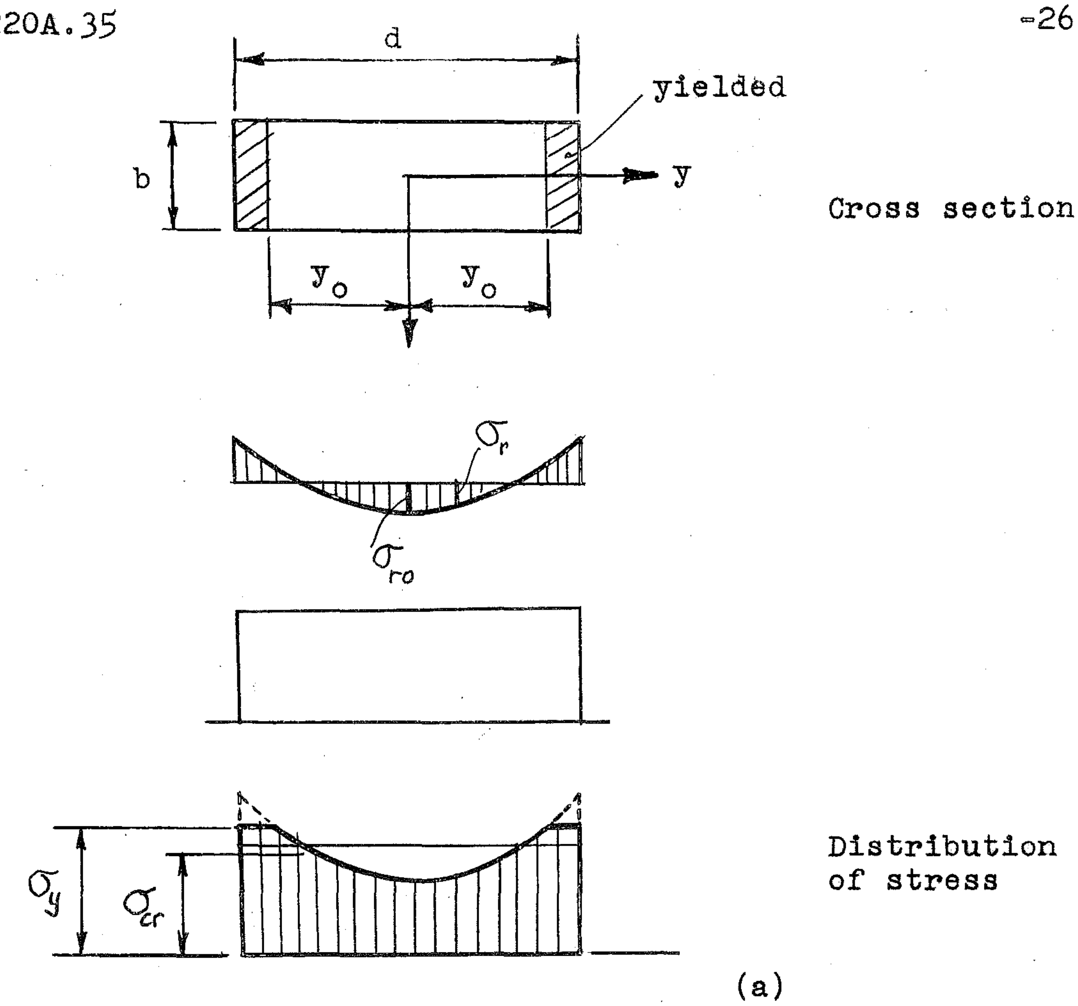


Fig. 4

NOMENCLATURE
FOR
RESIDUAL STRESS DISTRIBUTION
IN H-SHAPE



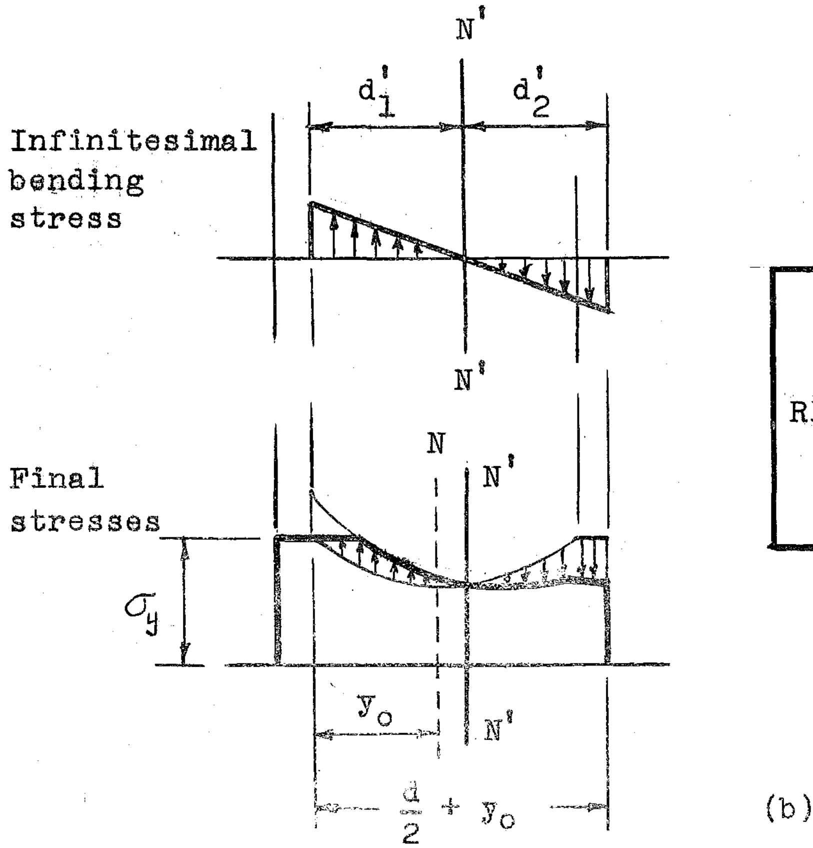


Fig. 5

REDUCED MODULUS CONCEPT.
YIELDING OF THE
CROSS SECTION

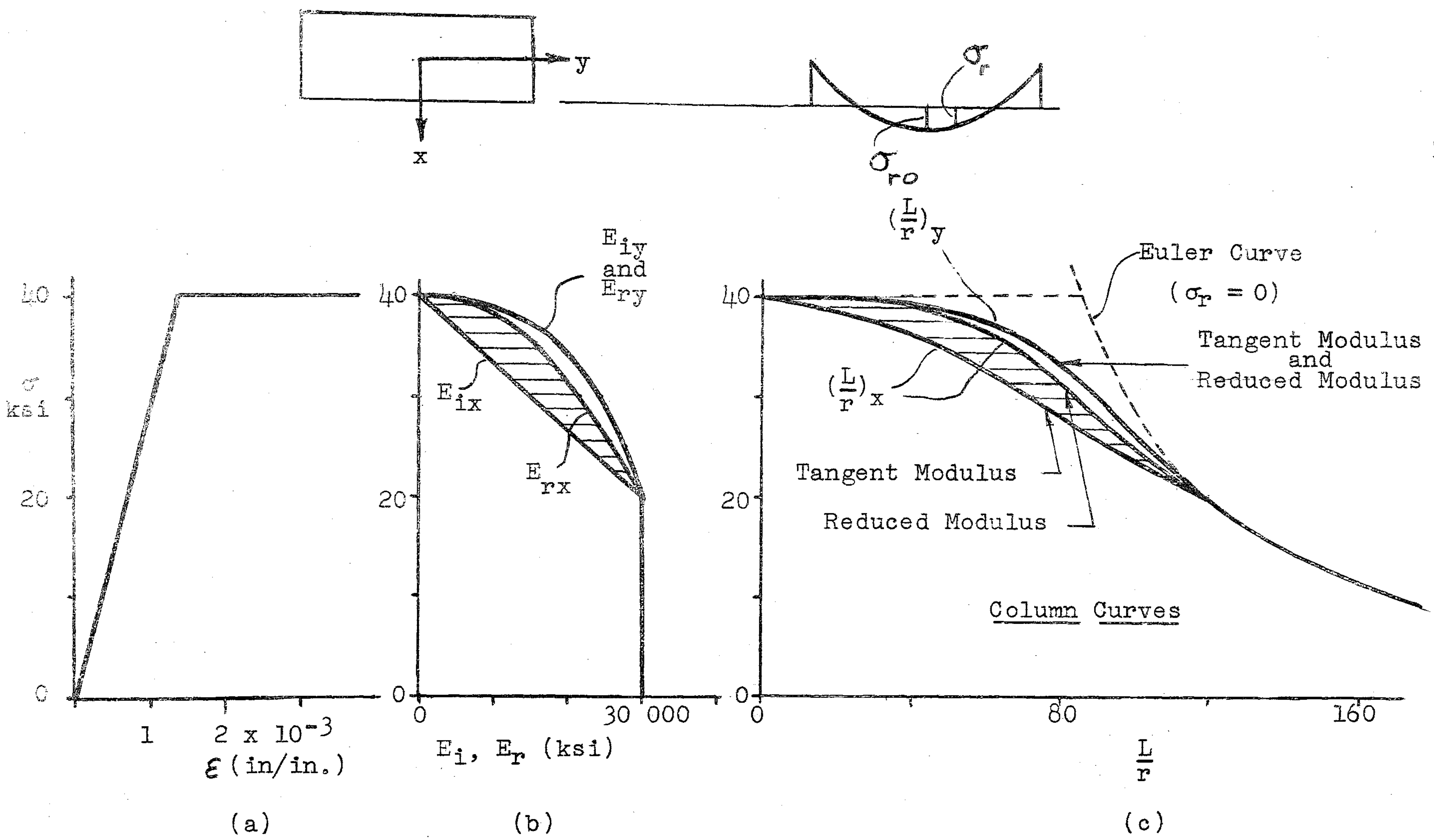


Fig. 6: IDEALIZED ELASTIC-PLASTIC STRESS-STRAIN RELATIONSHIP

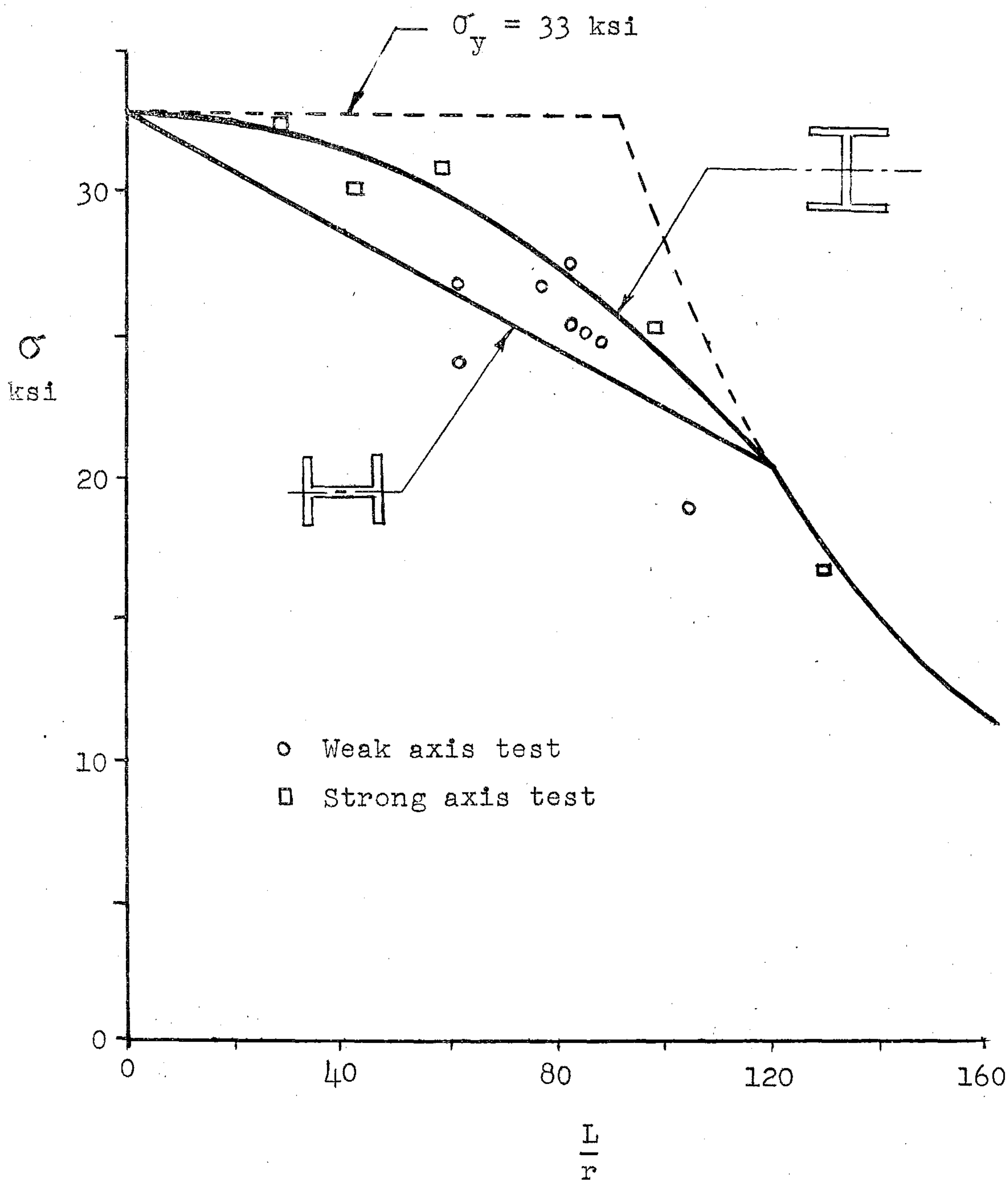


Fig. 7

BASIC COLUMN CURVES

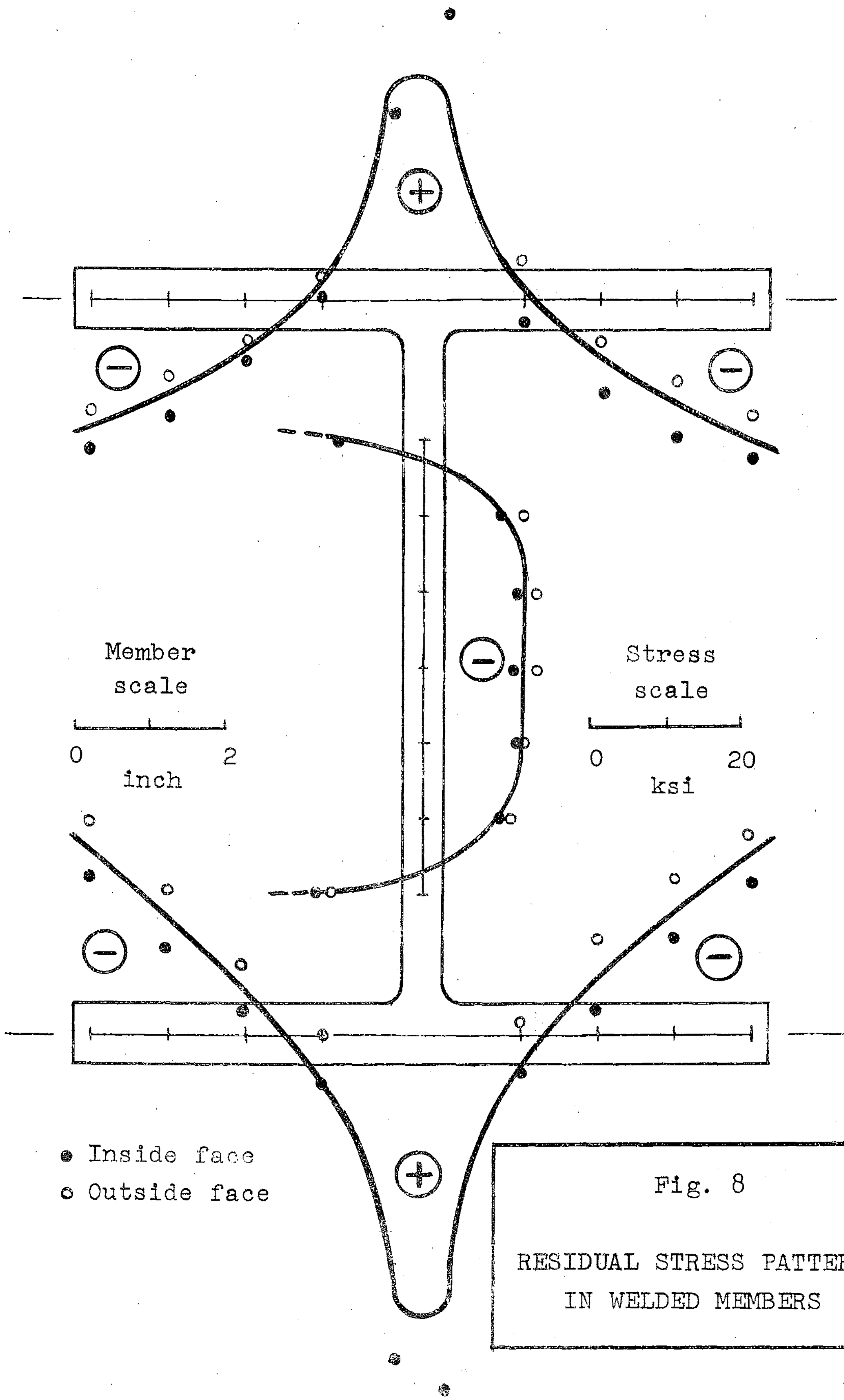


Fig. 8

RESIDUAL STRESS PATTERN IN WELDED MEMBERS

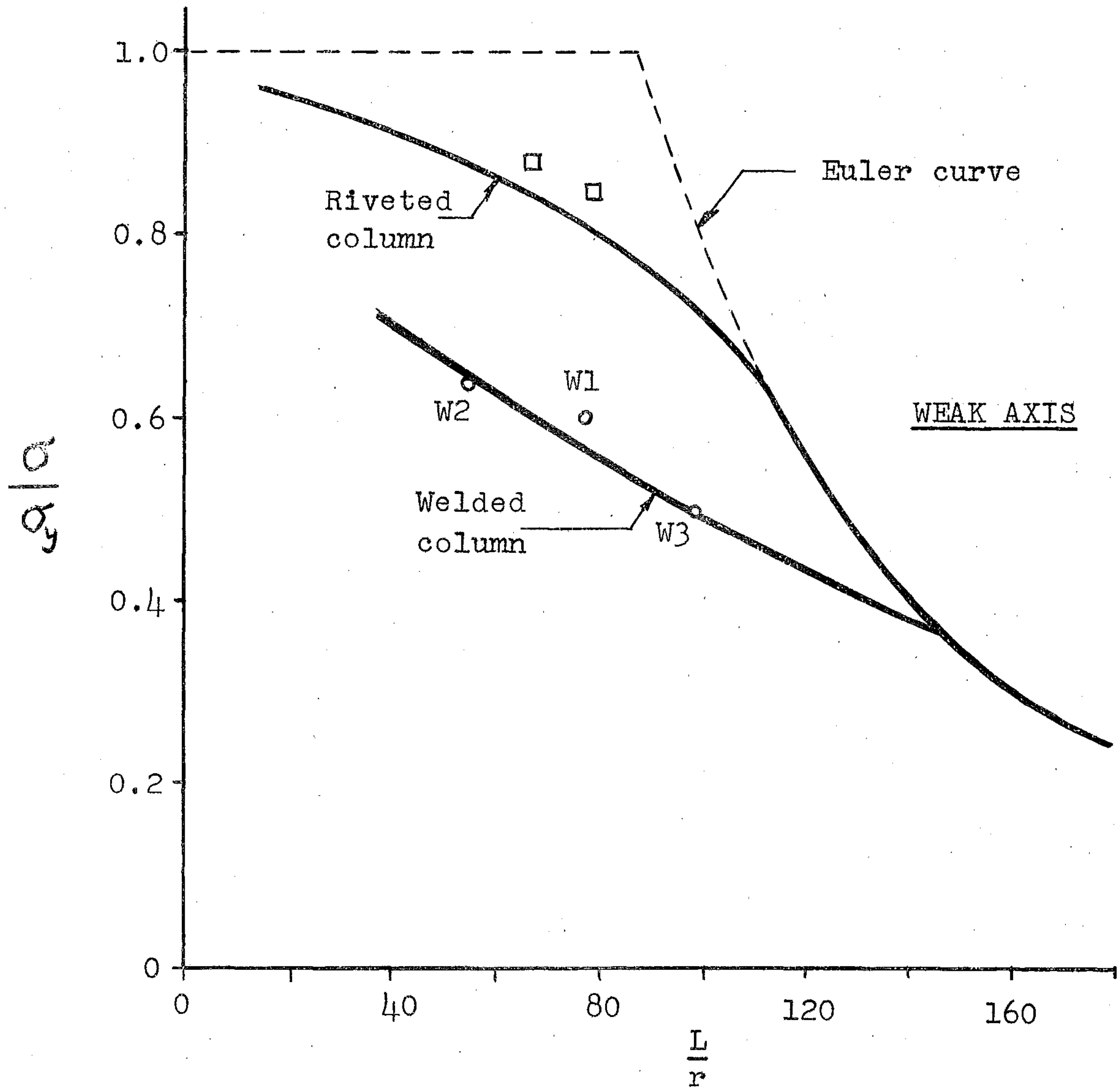


Fig. 9

BUILT-UP COLUMNS, TEST RESULTS

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